

### Extra Example 7

Find the coefficient of  $x^4$  in  $(2x - y)^7$ .  
280

### Key Questions to Ask for Example 7

- How do you know that the term containing  $x^4$  occurs when  $r = 6$ ?  
Each term is  ${}_{10}C_r(3x)^{10-r}(2)^r$ . So if the exponent  $10 - r$  is 4, then  $r = 6$ .
- Why is 210 a factor of the coefficient of  $x^4$ ?  ${}_{10}C_6 = 210$
- What is the sixth value in the tenth row of Pascal's triangle?  
210

### Closing the Lesson

Have students summarize the major points of the lesson and answer the Essential Question: How can you determine the value of  ${}_nC_r$  besides applying the formula?

- The formula for the number of combinations of  $r$  objects taken from a group of  $n$  distinct objects is  ${}_nC_r = \frac{n!}{(n-r)! \cdot r!}$ .
- Multiply when counting the number of ways two events can occur together. Add when counting the number of ways that either one event or another event can occur.
- The binomial theorem provides a technique for expanding a binomial or finding the coefficient of a particular term in the expansion.

You can use Pascal's triangle to determine  ${}_nC_r$ . In Pascal's triangle, the  $r$ th value in the  $n$ th row is  ${}_nC_r$ .

### EXAMPLE 7 Find a coefficient in an expansion

Find the coefficient of  $x^4$  in the expansion of  $(3x + 2)^{10}$ .

#### Solution

From the binomial theorem, you know the following:

$$(3x + 2)^{10} = {}_{10}C_0(3x)^{10}(2)^0 + {}_{10}C_1(3x)^9(2)^1 + \cdots + {}_{10}C_{10}(3x)^0(2)^{10}$$

Each term in the expansion has the form  ${}_{10}C_r(3x)^{10-r}(2)^r$ . The term containing  $x^4$  occurs when  $r = 6$ :

$${}_{10}C_6(3x)^4(2)^6 = (210)(81x^4)(64) = 1,088,640x^4$$

► The coefficient of  $x^4$  is 1,088,640.



#### GUIDED PRACTICE for Example 7

- Find the coefficient of  $x^5$  in the expansion of  $(x - 3)^7$ . 189
- Find the coefficient of  $x^3$  in the expansion of  $(2x + 5)^8$ . 1,400,000

## 10.2 EXERCISES

### HOMEWORK KEY

- = WORKED-OUT SOLUTIONS on p. WS1 for Exs. 17, 29, and 49
- ★ = STANDARDIZED TEST PRACTICE Exs. 2, 35, 40, 41, and 52

### SKILL PRACTICE

11. The denominator should have been multiplied by 2!;

$$\frac{6!}{(6-2)! \cdot 2!} = \frac{720}{48} = 15.$$

EXAMPLES 1, 2, and 3 on pp. 690–691 for Exs. 3–18

12. The denominator should have been multiplied by  $(8-3)!$ ;

$$\frac{8!}{(8-3)! \cdot 3!} = \frac{40,320}{720} = 56.$$

1. **VOCABULARY** Copy and complete: The binomial expansion of  $(a + b)^n$  is given by the ?.  *$n$ th row of Pascal's triangle*

2. **★ WRITING** Explain the difference between permutations and combinations. *In a permutation the order of the events is important, but in a combination the order is not important.*

**COMBINATIONS** Find the number of combinations.

- ${}_5C_2$  10
- ${}_{10}C_3$  120
- ${}_9C_6$  84
- ${}_8C_2$  28
- ${}_{11}C_{11}$  1
- ${}_{12}C_4$  495
- ${}_7C_5$  21
- ${}_{14}C_6$  3003

**ERROR ANALYSIS** Describe and correct the error in finding the number of combinations.

11.  ${}_6C_2 = \frac{6!}{(6-2)!} = \frac{720}{24} = 30$  ✗

12.  ${}_8C_3 = \frac{8!}{3!} = \frac{40,320}{6} = 6720$  ✗

**CARD HANDS** Find the number of possible 5-card hands that contain the cards specified. The cards are taken from a standard 52-card deck.

- 5 face cards (kings, queens, or jacks) 792 hands
- 4 kings and 1 other card 48 hands
- 1 ace and 4 cards that are not aces 778,320 hands
- 5 hearts or 5 diamonds 2574 hands
- At most 1 queen 2,490,624 hands
- At least 1 spade 2,023,203 hands



- $x^6 + 18x^5 + 135x^4 + 540x^3 + 1215x^2 + 1458x + 729$
- $y^6 - 18y^5z + 135y^4z^2 - 540y^3z^3 + 1215y^2z^4 - 1458yz^5 + 729z^6$
- $a^8 + 8a^7b^2 + 28a^6b^4 + 56a^5b^6 + 70a^4b^8 + 56a^3b^{10} + 28a^2b^{12} + 8ab^{14} + b^{16}$
- $128s^7 - 448s^6t^4 + 672s^5t^8 - 560s^4t^{12} + 280s^3t^{16} - 84s^2t^{20} + 14st^{24} - t^{28}$

**EXAMPLE 4**  
on p. 692  
for Exs. 19–23

**EXAMPLES 5 and 6**  
on p. 693  
for Exs. 24–31

**EXAMPLE 7**  
on p. 694  
for Exs. 32–35

36. row 0: 1,  
row 1: 2, row 2:  
4, row 3: 8,  
row 4: 16; row  
 $n$ :  $2^n$

37. The sum  
along each  
diagonal  
segment is equal  
to the sum of the  
two previous  
diagonal  
segment sums.

19. **USING PATTERNS** Copy Pascal's triangle on page 692 and add rows for  $n = 6, 7, 8, 9$ , and 10. **See margin.**

**PASCAL'S TRIANGLE** Use the rows of Pascal's triangle from Exercise 19 to write the binomial expansion. **20–23. See margin.**

20.  $(x + 3)^6$       21.  $(y - 3z)^{10}$       22.  $(a + b^2)^8$       23.  $(2s - t^4)^7$

**BINOMIAL THEOREM** Use the binomial theorem to write the binomial expansion. **24–31. See margin.**

24.  $(x + 2)^3$       25.  $(c - 4)^5$       26.  $(a + 3b)^4$       27.  $(4p - q)^6$   
28.  $(w^3 - 3)^4$       **29.**  $(2s^4 + 5)^5$       30.  $(3u + v^2)^6$       31.  $(x^3 - y^2)^4$

32. Find the coefficient of  $x^5$  in the expansion of  $(x - 2)^{10}$ . **-8064**

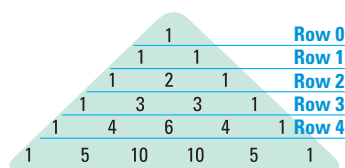
33. Find the coefficient of  $x^3$  in the expansion of  $(3x + 2)^5$ . **1080**

34. Find the coefficient of  $x^6$  in the expansion of  $(x^2 - 3)^8$ . **-13,608**

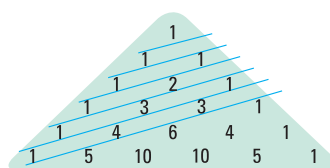
35. **★ MULTIPLE CHOICE** Which is the coefficient of  $x^4$  in the expansion of  $(x - 3)^7$ ? **A**  
(A) -945      (B) -35      (C) -27      (D) 2835

**B PASCAL'S TRIANGLE** In Exercises 36 and 37, use the diagrams shown.

36. What is the sum of the numbers in each of rows 0–4 of Pascal's triangle? What is the sum in row  $n$ ?



37. Describe the pattern formed by the sums of the numbers along the diagonal segments of Pascal's triangle.



**REASONING** In Exercises 38 and 39, decide whether the problem requires combinations or permutations to find the answer. Then solve the problem.

38. **NEWSPAPER** Your school newspaper has an editor-in-chief and an assistant editor-in-chief. The staff of the newspaper has 12 students. In how many ways can students be chosen for these two positions? **permutations; 132 ways**

39. **STUDENT COUNCIL** Five representatives from a senior class of 280 students are to be chosen for the student council. In how many ways can students be chosen to represent the senior class on the student council?  
**combinations; 13,836,130,056 ways**

40. **★ MULTIPLE CHOICE** A relay race has a team of 4 runners who run different parts of the race. There are 20 students on your track squad. In how many ways can the coach select students to compete on the relay team? **C**

(A) 4845      (B) 40,000      (C) 116,280      (D) 160,000

41. **★ SHORT RESPONSE** Explain how the formula for  ${}_nC_n$  suggests the definition  $0! = 1$ .  **$1 = {}_nC_n = \frac{n!}{(n-n)!n!} = \frac{n!}{0!n!} = \frac{1}{0!}$ , so  $0!$  must equal 1.**

**C CHALLENGE** Verify the identity. Justify each of your steps. **42–47. See margin.**

42.  ${}_nC_0 = 1$

43.  ${}_nC_n = 1$

44.  ${}_nC_r \cdot {}_rC_m = {}_nC_m \cdot {}_{n-m}C_{r-m}$

45.  ${}_nC_1 = {}_nP_1$

46.  ${}_nC_r = {}_nC_{n-r}$

47.  ${}_{n+1}C_r = {}_nC_r + {}_nC_{r-1}$

24.  $x^3 + 6x^2 + 12x + 8$

25.  $c^5 - 20c^4 + 160c^3 - 640c^2 + 1280c - 1024$

26.  $a^4 + 12a^3b + 54a^2b^2 + 108ab^3 + 81b^4$

27.  $4096p^6 - 6144p^5q + 3840p^4q^2 - 1280p^3q^3 + 240p^2q^4 - 24pq^5 + q^6$

28.  $w^{12} - 12w^9 + 54w^6 - 108w^3 + 81$

29.  $32s^{20} + 400s^{16} + 2000s^{12} + 5000s^8 + 6250s^4 + 3125$

30.  $729u^6 + 1458u^5v^2 + 1215u^4v^4 + 540u^3v^6 + 135u^2v^8 + 18uv^{10} + v^{12}$

31.  $x^{12} - 4x^9y^2 + 6x^6y^4 - 4x^3y^6 + y^8$

## 4 PRACTICE AND APPLY

### Assignment Guide

**Answer Transparencies**  
available for all exercises

**Basic:**

Day 1: pp. 694–697

Exs. 1–5, 11–14, 19–21, 24–26,  
32–37, 48–51, 54–66 even

**Average:**

Day 1: pp. 694–697

Exs. 1, 2, 6–8, 11, 12, 15, 16, 19, 21,  
22, 27–29, 32–41, 48–52, 55, 59, 65

**Advanced:**

Day 1: pp. 694–697

Exs. 1, 2, 9, 10, 17–19, 22, 23,  
30–47\*, 49–53\*, 57, 63, 67

**Block:**

pp. 694–697

Exs. 1, 2, 6–8, 11, 12, 15, 16, 19, 21,  
22, 27–29, 32–41, 48–52, 55, 59, 65  
(with 10.1)

### Differentiated Instruction

See *Algebra 2 Best Practices Toolkit*  
for suggestions on addressing the  
needs of a diverse classroom.

### Homework Check

For a quick check of student under-  
standing of key concepts, go over  
the following exercises:

**Basic:** 4, 14, 20, 26, 48

**Average:** 8, 16, 22, 28, 49

**Advanced:** 10, 18, 30, 34, 50

### Extra Practice

• Student Edition, p. 1019

• Chapter 10 Resource Book:  
Practice levels A, B, C, pp. 18–20

### Practice Worksheet

**An easily-readable reduced  
practice page (with answers)  
for this lesson can be found  
on p. 680C.**

**42–47. See Additional Answers**  
beginning on p. AA1.